

Approximation with Hierarchical Low Rank Tensors

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The first part of the lecture series will provide a general introduction to low rank tensor formats for the data sparse representation of higher order tensors and multivariate functions. The classical variants are the Canonical Polyadic (CP) format, the Tucker format, Tensor Trains (TT) and Hierarchical Tucker (HT) representations. Each of the formats comes with its own notion of rank in higher dimension, and all of them collapse to the same classical matrix rank in two dimensions.

Each of the notions of rank gives rise to a different set or manifold of tensors of fixed rank and we compare the advantages and drawbacks between the different formats. The concepts are introduced in the discrete setting where a tensor is a mapping from a Cartesian product of finite index sets, but we also point out the relation to multivariate functions, the continuous setting. We summarize some interesting open questions that open new and sometimes very difficult areas of research.

The second half of the lectures is geared towards more recent results on stability and approximability – which is quite limited, since classical notions of smoothness do not characterize the relevant classes of low rank tensors. Finally, we broaden the viewpoint from low rank tensor formats in the direction of tensor networks as well as neural networks. estimates.