Sparse Adaptive Smolyak Quadrature Approach to Bayesian Inverse Problems

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We present a deterministic, computational approach to a broad class of inverse problems for identification of distributed parameters in ordinary or partial differential equations, given gaussian noisy measurements.

Based on the parametric deterministic formulation of Bayesian inverse problems with unknown input parameter from infinite dimensional, separable Banach spaces, and uniform prior on the parameter uncertainty, we develop a practical, adaptive computational algorithm for the efficient approximation of the infinite-dimensional integrals with respect to the posterior measure which arise in Bayesian estimation.

Convergence rates for the adaptive Smolyak quadrature approximation are shown, computationally, to coincide with the best N term approximation rates of the Bayesian posterior density which, in turn, are proved [1, 5, 4] to depend only on the sparsity class of the uncertain distributed parameter, but which are bounded independently of the number of uncertain parameters which are activated by the adaptive algorithm. The analysis is based on analytic continuation and is applicable for wide classes of rPDEs with uncertain coefficients and domains of definition. They are, in particular, higher than those of MCMC methods in terms of the number M of solutions of the forward problems. Applications for high-dimensional parametric initial value problems from biological systems sciences are presented.

References

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