PARAMETRIC INFERENCE FOR DISCRETE OBSERVATIONS OF DIFFUSIONS WITH RANDOM EFFECTS IN THE DRIFT AND IN THE DIFFUSION COEFFICIENT

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Abstract

We consider N independent stochastic processes $(X_i(t), t \in [0,T])$, $i = 1, \ldots, N$, defined by a stochastic differential equation with drift term depending on a random vecteur Φ_i and diffusion coefficient depending on another random effect Ψ_i:

$$dX_i(t) = \Phi_i' b(X_i(t))dt + \Psi_i \sigma(X_i(t))\, dW_i(t), \quad X_i(0) = x,\ i = 1, \ldots, N,$$

where $(W_1, \ldots, W_N)$ are N independent Wiener processes, $((\Phi_i, \Psi_i), i = 1, \ldots N)$ are N i.i.d. $\mathbb{R}^d \times (0, +\infty)$-valued random variables, $((\Phi_i, \Psi_i), i = 1, \ldots, N)$ and $(W_1, \ldots, W_N)$ are independent and x is a known real value. The functions $\sigma(.) : \mathbb{R} \to \mathbb{R}$ and $b(.) = (b_1(.), \ldots, b_d(.))' : \mathbb{R} \to \mathbb{R}^d$ are known. Each process $(X_i(t))$ represents an individual and the $d+1$-dimensional random vector (Φ_i, Ψ_i) represents the random effect of individual i and is discretely observed on a fixed-length time interval $[0,T]$ with $T > 0$ at n times $t_j = jT/n$. We consider parametric distributions for the random effects and estimate the unknown parameters from the observations $\{X_i(t_j), j = 1, \ldots, n, i = 1, \ldots, N\}$, as n, N go to infinity. We study a case that gives rise to an explicit approximation of the likelihood function:

- (Φ_i, Ψ_i) has the following distribution:

$$\Psi_i = \frac{1}{\Gamma_i^{1/2}}, \quad \Gamma_i \sim G(a, \lambda), \text{ and given } \Gamma_i = \gamma, \quad \Phi_i \sim \mathcal{N}_d(\boldsymbol{\mu}, \gamma^{-1}\boldsymbol{\Omega}).$$

We characterize the asymptotic behaviour of the associated estimators.

References

[1] Delattre M., Genon-Catalot V., Larédo C. (2016) *Parametric inference for discrete observations of diffusions with random effects in the drift or in the diffusion coefficient*,
[2] Delattre M., Genon-Catalot V., Larédo C. (2016) *Parametric inference for discrete observations of diffusions with random effects in the drift and in the diffusion coefficient*.