ON MISSPECIFICATION IN REGULARITY AND PROPERTIES OF ESTIMATORS

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Parameter estimation; Misspecification in regularity; Cusp and change-point singularities; Asymptotic properties:

The observed process $X^T = (X_t, 0 \le t \le T)$ satyisfies the equation

$$\mathrm{d}X_t = S(\vartheta_0, t)\,\mathrm{d}t + \varepsilon \mathrm{d}W_t, \qquad X_0 = 0, \qquad 0 \le t \le T, \tag{1}$$

where $S(\vartheta, t)$ is the signal and $W_t, 0 \le t \le T$ is a Wiener process. The statistician supposes that the observed (theoretical) signal is $Q(\vartheta, t)$ and the model is

$$\mathrm{d}X(t) = Q(\vartheta, t)\,\mathrm{d}t + \varepsilon \mathrm{d}W_t, , \qquad X_0 = 0, \qquad 0 \le t \le T. \tag{2}$$

Therefore we have to estimate ϑ_0 of the equation (1) using the model (2) and observations (1). Hence we are in the situation of *misspecification*. We are mainly interested by the estimation of ϑ_0 in the cases where the regularity conditions (smoothness) of the signals $S(\vartheta, t)$ and $Q(\vartheta, t)$ are different. For example, in change-point type problems it is supposed that the observed signal is discontinuous in time, but the real technical device can not provide a signal of such form and the real signals can be continuous or cusp-type close to the (theoretical) discontinuous signal. The asymptotic corresponds to $\varepsilon \to 0$, i.e., we have *small noise* case.

We consider several statements

1. Signal $S(\vartheta, t)$ is smooth. Signal $Q(\vartheta, t)$ is close discontinuous (change-point problem).
2. Signal $S(\vartheta, t)$ is discontinuous (change-point problem). Signal $Q(\vartheta, t)$ is close continuous.
3. Signal $S(\vartheta, t)$ is of cusp-type. Signal $Q(\vartheta, t)$ is close continuous.
4. Signal $S(\vartheta, t)$ is of cusp-type. Signal $Q(\vartheta, t)$ is close discontinuous.
5. Signal $S(\vartheta, t)$ is discontinuous. Signal $Q(\vartheta, t)$ is of cusp-type close.
6. Signal $S(\vartheta, t)$ is smooth. Signal $Q(\vartheta, t)$ is of cusp-type close.

etc.

In all problems we describe the asymptotic behavior of the MLE estimators. For example, in the cases 1 and 6 we have

$$\frac{\hat\vartheta_\varepsilon - \vartheta_0}{\varepsilon^{\frac{2}{3}}} \Longrightarrow \zeta_1, \qquad\qquad \frac{\hat\vartheta_\varepsilon - \vartheta_0}{\varepsilon^{\frac{2}{3-2\kappa}}} \Longrightarrow \zeta_2$$

respectively. Here ζ_1 and ζ_2 are two random variables expressed as some functionals of the Wiener and fBm processes.

References

[1] Chernoyarov, O.V., Kutoyants, Yu.A., Trifonov, A.P. (2015) *On misspecification in regularity and properties of estimators*. Submitted.

[2] Chernoyarov, O.V., Dachian, S.Yu., Kutoyants, Yu.A. (2015) *On prameter estimation for cusp-type signal*. Submitted.