SECOND ORDER ASYMPTOTIC EFFICIENCY FOR A POISSON PROCESS

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Observations are i.i.d. inhomogeneous Poisson processes $\mathbf{X} = (X_1, \cdots, X_n)$, on a bounded interval $[0, \tau]$ of the real line, $X_j = \{X_j(t),\, t \in [0, \tau]\}$. We are interested in the estimation problem of the mean function $\mathbf{E}X_j(t) = \Lambda(t)$ of this process. Kutoyants proved that the empirical mean function $\hat{\Lambda}_n(t) = \frac{1}{n}\sum_{j=1}^{n} X_j(t)$ is an asymptotically efficient estimator with respect to the mean integrated square error (MISE), in the minimax sense. We start by describing a class of kernel-type estimators that are also asymptotically efficient. We call these estimators first order efficient estimators. To choose the best one among them, we prove a lower bound that compares the second order term of the MISE of all estimators. We find the optimal rate of convergence of the second order term of the MISE, which depends on the smoothness of the unknown mean function. We calculate also explicitly the asymptotic minimal value of the second order term, which plays the same role as the Pinsker constant in the Pinsker's minimax theory. In the end, we construct an estimator which attains this lower bound, thus is asymptotically second order efficient. The notion of the second order efficiency was introduced by Golubev and Levit. Minimax optimization is done over Sobolev ellipsoids, following the ideas of M.S. Pinsker.

References

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