Mixed normal-superconducting states in the presence of strong electric currents

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Onnes (1911), Meissner & Ochsenfeld (1933)

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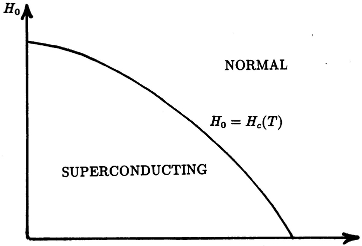

FIG. 1. *Critical field as a function of temperature.*

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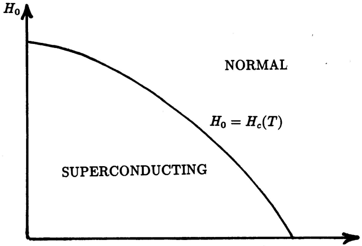

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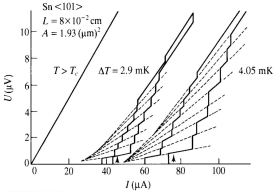
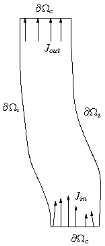
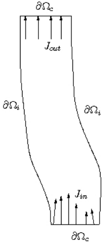


FIGURE 11.11
Current-voltage characteristics of tin "whiskers" showing regular step structures due to successive establishment of phase-slip centers. Here $\Delta T = T_c - T$. (*After Meyer.*)



$\partial\Omega_c$
J_{out}
$\partial\Omega_i$
$\partial\Omega_i$
J_{in}
$\partial\Omega_c$



$$\frac{\partial \psi_\kappa}{\partial t} - \nabla^2_{\kappa A_\kappa}\psi_\kappa + i\kappa\phi_\kappa\psi_\kappa = \kappa^2(1-|\psi_\kappa|^2)\psi_\kappa \qquad \text{in } (0,+\infty)\times\Omega\,,$$

$$\frac{\partial A_\kappa}{\partial t} + \nabla\phi_\kappa + \text{curl}^2 A_\kappa = \frac{1}{\kappa}\Im(\bar{\psi}_\kappa\nabla_{\kappa A_\kappa}\psi_\kappa) \qquad \text{in } (0,+\infty)\times\Omega\,,$$

$$\psi = 0 \qquad \text{on } (0,+\infty)\times\partial\Omega_c\,,$$

$$\nabla_{\kappa A_\kappa}\psi_\kappa\cdot\nu = 0 \qquad \text{on } (0,+\infty)\times\partial\Omega_i\,,$$

$$\frac{\partial \phi_\kappa}{\partial \nu} = -\kappa J(x) \qquad \text{on } (0,+\infty)\times\partial\Omega\,,$$

$$\int_{\partial\Omega}\text{curl}\,A_\kappa\,ds = \kappa h_{ex} \qquad \text{on } (0,+\infty)\,,$$

Gauge invariance

$$A \to A + \nabla\omega \quad \phi \to \phi - \frac{\partial\omega}{\partial t} \quad \psi \to \psi e^{i\omega}$$

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Magnetic field on the boundary

$$J|_{\partial\Omega_i} = 0 \Rightarrow B_\kappa|_{\partial\Omega_{i,j}} \stackrel{\text{def}}{=} \operatorname{curl} A_\kappa|_{\partial\Omega_{i,j}} = h_j\,\kappa \quad J|_{\partial\Omega_c} \neq 0 \quad j \in \{1,2\}$$

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$$h_1 h_2 < 0\,,$$

$h = \max(|h_1|, |h_2|)$

$$1 < h \leq \frac{1}{\Theta_0} \quad \text{or} \quad \frac{1}{\Theta_0} < h \ll \kappa$$

$\Theta_0 \sim 0.59$ de Gennes' value

Linearized equation

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Normal fields: $(\psi, A, \phi) = (0, \kappa A_n, c\kappa\phi_n)$ $c = \kappa^2/\sigma$

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$$\mathcal{L}_\kappa = \left(i\nabla + \kappa^2 A_n\right)^2 + ic\kappa^2\phi_n$$

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$$\mu(\kappa, J) \stackrel{def}{=} \inf_{\lambda\in\sigma(\mathcal{L}_\kappa)} \Re\lambda < 1 \Rightarrow u \equiv 0 \text{ unstable}$$

$J = J_0 \kappa \Rightarrow h \sim \mathcal{O}(\kappa)$

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$D(\mathcal{A}^+) = \{u \in H_0^{1,\mathrm{mag}}(\mathbb{R}^2_+, \mathbb{C}) \cap L^2(\mathbb{R}^2_+, \mathbb{C}; y\, dxdy) \,:\, \mathcal{A}_+ u \in L^2(\mathbb{R}^2_+, \mathbb{C})\}$

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Theorem

Let $\mu_\infty = \liminf_{\kappa\to\infty} \mu$. *Then*

$$\limsup_{\kappa\to\infty} \mu = \mu_\infty = \nu_m\,.$$

Almog, Helffer (2014)

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$$K \cap \partial\Omega_i \neq \emptyset \Rightarrow \kappa^2\Theta_{\pi/2}^{DN} \inf_{x\in K} |B_n(x)|\,\|\eta\psi_\kappa\|_2^2 \leq \|\nabla_{\kappa^2 A_n}(\eta\psi_\kappa)\|_2^2$$

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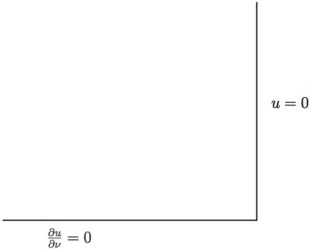
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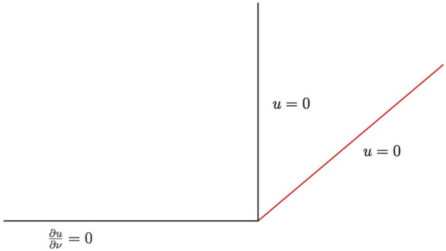
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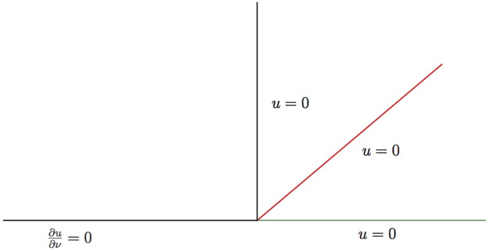
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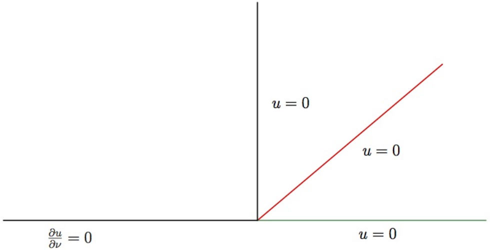

$$\Theta_0 \geq \Theta_\alpha^{DN} \geq \Theta_\pi^{DN} \geq \Theta_0$$

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$$E_{min} \sim \kappa^2 \left[\frac{1}{2} + \int_\Omega g\left(B_n(x)\right) dx\right]$$

$-1/2 < g(b) \leq g(1) = 0$ increasing - Sandier & Serfaty (2003)

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$$\begin{cases} -\Delta\phi_\kappa + c\,|\psi_\kappa|^2\phi_\kappa = 0 & \text{in } \Omega \\ \frac{\partial\phi_\kappa}{\partial\nu} = c\kappa J & \text{on } \partial\Omega_c \\ \frac{\partial\phi_\kappa}{\partial\nu} = 0 & \text{on } \partial\Omega_i \end{cases}$$

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Theorem

For any compact set $K \subset \omega_j \cup \partial\Omega_c$ *there exist* $C > 0$, $\alpha > 0$, *and* $\kappa_0 \geq 1$, *such that for any* $\kappa \geq \kappa_0$ *we have*

$$\limsup_{t\to\infty} \int_K |\psi_\kappa(t,x)|^2 \, dx \leq Ce^{-\alpha\kappa} \,.$$

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$$\begin{aligned}
&\Delta_A\psi + \psi\left(1 - |\psi|^2\right) - i\phi\psi = 0 && \text{in } \Omega^R\,, \\
&\text{curl}^2 A + \nabla\phi = \Im(\bar{\psi}\,\nabla_A\psi) && \text{in } \Omega^R\,, \\
&\psi = 0 && \text{on } \partial\Omega_c^R\,, \\
&\nabla_A\psi\cdot\nu = 0 && \text{on } \partial\Omega_i^R\,, \\
&\frac{\partial\phi}{\partial\nu} = R^{-(1-\gamma)}J && \text{on } \partial\Omega^R\,, \\
&\fint_{\partial\Omega^R} \text{curl}\, A(x)\, ds = R^\gamma h_{ex}\,.
\end{aligned}$$

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&\text{curl}^2 A + \nabla\phi = \Im(\bar\psi\,\nabla_A\psi) && \text{in } \Omega^R\,, \\
&\psi = 0 && \text{on } \partial\Omega^R_c\,, \\
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&\frac{\partial\phi}{\partial\nu} = R^{-(1-\gamma)} J && \text{on } \partial\Omega^R\,, \\
&\fint_{\partial\Omega^R} \text{curl}\, A(x)\, ds = R^\gamma h_{ex}\,.
\end{aligned}$$

$$0 < \gamma < 1$$

Proposition

$\exists K \subset \Omega$ *compact*, $C > 0$, $R_0 > 0$, $\alpha > 0$:

$$R > R_0 \Rightarrow \int_{K_R} |\psi(x)|^2 \, dx \leq Ce^{-\alpha R} \,,$$

$K_R = T_R(K)$

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$$\begin{aligned}
&\Delta_{\epsilon^{-2} A_\epsilon} \psi_\epsilon + \frac{\psi_\epsilon}{\epsilon^2}\left(1 - |\psi_\epsilon|^2\right) - \frac{i}{\epsilon^2} \phi_\epsilon \psi_\epsilon = 0 && \text{in } \Omega \\
&\operatorname{curl}^2 A_\epsilon + \nabla \phi_\epsilon = \Im\left(\bar{\psi}_\epsilon \nabla_{\epsilon^{-2} A_\epsilon} \psi_\epsilon\right) && \text{in } \Omega \\
&\frac{\partial \phi_\epsilon}{\partial \nu} = \epsilon^{-\gamma} J && \text{on } \partial\Omega
\end{aligned}$$

$$A_{1,\epsilon} = A_\epsilon - \epsilon^{-\gamma} A_n \quad ; \quad B_{1,\epsilon} = \text{curl}\, A_{1,\epsilon}\,.$$

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Conjecture

For any compact set $K \subset \Omega \setminus B_n^{-1}(0)$, $\exists R_0 > 0,\ C > 0,$ *and* $\alpha > 0$

$$R > R_0 \Rightarrow \int_{K_R} |\psi(x)|^2 \, dx \leq Ce^{-\alpha R}$$

Handbook of Superconductivity

Poole (2000)

Table 9.6.

Critical current density J_c for various superconductors. Values for the temperature T_{meas} and the field B_{app} of measurement are also given. References are identified by first author and year.

Material	J_c (A/cm^2)	B_{app} (tesla)	T_{meas} (K)	Comments	Reference
Melt textured high-T_c materiels		0.02 to 1	77	$J_c \sim [1-(T/T_c)]^{\alpha}/B^{\beta}$, where $\alpha \leqslant 7$, $\beta \sim 0.5$	Fisher (1994)
Cu–Nb composite	2.5×10^5	Self field	4.2	$T_c = 8.8$ K	Reddy (1986)
NbTi	3.4×10^5	5	4.2	Superconducting composite	Li (1983)
	3×10^5	5		Filament diameters 5 to 9 μm	Kanithi (1989)
	3×10^5	0.5	4.2	Filament diameter 50–500 nm; see $\xi\lambda$ table	Cave (1989)
	1.41×10^6	1		Wire enclosed by CuNi alloy, 0.072 μm filament diameter, J_c is less for 0.061 μm filament diameter	Kumano (1990)
	1.07×10^6	1		0.4 μm filament, artificial pinning control	Miura (1992)
	4.44×10^5	3		0.4 μm filament, artificial pinning control	Miura (1992)
	5.2×10^5	5	4.2	47 wt% Ti, filaments are aligned	Cooley (1991)
	2.85×10^5	5		48 wt%Ti, 210 filament, 1.6 mm diameter wire	Chernyi (1997)
	3.5×10^5	5	4.2	Industrial wires	Chernoplekov (1992)
	3.36×10^5	3		Wire, 54 wt% Ti	Inoue (1995)
	4.6×10^5	5	4.2	Artificial pinning center wire, 47 wt%Ti	Heussner (1997)
NbN	2×10^4	0		Josephson junction 1.5 μm^2 area	Tarutani (1984)

(continued)